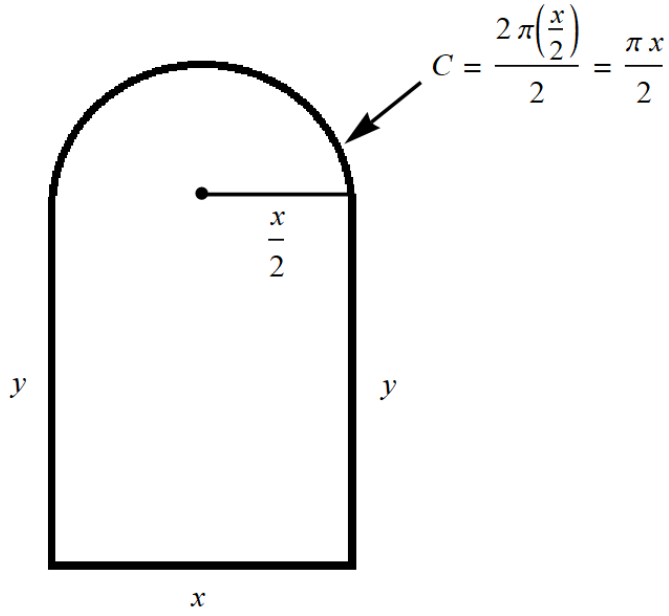
Exercise 62

A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 30 ft, express the area A of the window as a function of the width x of the window.

Solution



The perimeter consists of the semicircular arc, two times the vertical height y, and the horizontal distance x.

$$P = x + y + y + \pi\left(\frac{x}{2}\right) = 30$$

Solve this equation for y.

$$x + 2y + \frac{\pi x}{2} = 30$$

$$2y = 30 - x - \frac{\pi x}{2}$$

$$y = 15 - \frac{x}{2} - \frac{\pi x}{4} \tag{1}$$

The area of the window is the sum of the rectangle's and the half-circle's.

$$\begin{aligned} A &= xy + \frac{\pi\left(\frac{x}{2}\right)^2}{2} \\ &= xy + \frac{\pi\frac{x^2}{4}}{2} \\ &= xy + \frac{\pi x^2}{8} \end{aligned}$$

Substitute equation (1) for y so that A is a function of x only.

$$\begin{aligned} A(x) &= x\left(15 - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{\pi x^2}{8} \\ &= 15x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8} \\ &= 15x - \left(\frac{1}{2} + \frac{\pi}{4} - \frac{\pi}{8}\right)x^2 \\ &= 15x - \left(\frac{1}{2} + \frac{\pi}{8}\right)x^2 \\ &= 15x - \frac{1}{8}(4+\pi)x^2 \end{aligned}$$